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ANALYSIS OF A BROADBAND CERENKOV AMPLIFIER.(U)

MAR 80 J VOMVORIDIS, R SMITH N00014-78-C-0613

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ANALYSIS OF A BROADBAND CERENKOV AMPLIFIER

JAYCOR Project 6101
Contract No. N00014-78-C-0613
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--  24 - APPROACH·  (U)  DIELECTRICALLY LOADED GYROTRON TRAVELING WAVE AMPLFIER
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--     EFFICIENCIES. EXPERIMENTAL STRUCTURES WILL BE DESIGNED, FABRICATED AND
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I. INTRODUCTION

In the past several years, there has been widespread interest in a variety of wave generation devices based on un-neutralized beams of relativistic or mildly relativistic electrons. Such devices are potentially capable of producing high power, with high efficiency, in the centimeter, millimeter, and sub-millimeter spectral ranges. An example is the electron cyclotron maser, which has produced $\approx$100 kW at centimeter wavelengths, with efficiencies ranging from 30% - 50%. The gyrotron mechanism involves azimuthal phase bunching of the electron beam, which occurs during resonant interaction of the Doppler-shifted beam cyclotron mode with a waveguide or cavity mode ~~(Fig. 1)~~. The bandwidth of the gyrotron mechanism is typically of order 1-2%.

For various military and communications applications, it could be of great interest to develop free-electron amplifiers with wide operating bandwidths, ~~$\Delta\omega \gtrsim \omega$~~. In this report, we present a linear analysis of a new principle for wideband amplification. The basic configuration is the use of an annular electron beam drifting in a dielectrically-loaded waveguide ~~(Fig. 2)~~. In such a waveguide, the normal guide modes have phase velocities asymptotic to $c/\sqrt{\varepsilon}$, where c is the speed of light ~~and $\varepsilon > 1$ is~~ the dielectric constant (assumed to be isotropic) of the waveguide lining. The system is assumed to be immersed in an axial magnetic field, $\vec{B}_0$, sufficiently strong to render the motion of the beam electrons one-dimensional. In this case,

the amplification mechanism depends on axial charge bunching in the Cerenkov interaction of a longitudinal space-charge wave of the beam with the E_z - componant of a TM guide mode.

The interaction we envision is schematically represented in the dispersion diagrams of Fig. 3, where

$$\omega_{\pm} = kV \pm \frac{\omega_p}{\sqrt{\varepsilon\gamma^3}} \tag{1}$$

are the positive and negative energy beam modes and

$$\omega_g = \pm \frac{1}{\varepsilon} \sqrt{(kc)^2 + \left(\frac{2.4c}{r_4}\right)^2} \tag{2}$$

are the waveguide modes for the case of a constant isotropic dielectric material filling the waveguide. As can be seen in Fib. 3a, where $\varepsilon=1$, the beam and guide modes do not cross and therefore four distinct propagating modes are expected on the entire k spectrum, with their dispersion characteristics given by slight modification of Eqs (1) and (2) due to coupling. On the other hand however, Fig. 3b, when $\varepsilon \neq 1$ and in particular when $\varepsilon > (c/V)^2$ the dispersion curves do cross and therefore growing modes are expected due to the interaction. Most interesting appears to be the case when ε slightly exceeds the value $(c/V)^2$, since then strong interaction is expected over a broad band of frequencies.

The above discussion applies also to the topologically equivalent

case of a waveguide with partial dielectric filling, for which the dispersion curve is represented by the broken line in Fig. 3b. Since this configuration is more compatible with $\varepsilon \gtrsim (c/V)^2$, the analysis of this report is based on the realistic geometry of Fig. 2.

In Section II, we present the general linear analysis of this system, culminating in the dispersion relation of Eq.(11). In Section III the dispersion relation is solved analytically in a simple limiting case. In Section IV, we present the conclusions of this analysis and suggestions for future work on this concept.

II. ANALYSIS

A. Physical Model

The configuration and geometrical parameters are defined in Figure 2. The waveguide is a perfectly conducting sylinder of radius r_4. The guide is lined with a lossless dielectric (of isotropic dielectric constant ε) in the space $r_3 \leq r \leq r_4$. The electron beam is annular, filling the region $r_1 \leqslant r \leqslant r_2$. The regions $0 \leqslant r < r_1$, $r_2 < r < r_3$ are in vacuum. An applied axial magnetic field $B_o = B_o \hat{e}_z$ pervades the waveguide. The un-neutralized electron beam has purely axial motion; this corresponds to the limit $B_o \to \infty$, so that transverse motion is suppressed and no cyclotron interaction may occur. We assume a cold uniform annular beam, the equilibrium distribution function of which is given by

$$f_o\ (r,p_z)\ = n_o H\ (r-r_1)\ H\ (r_2-r)\ \delta\ (p_z - \gamma_o mV) \tag{3}$$

where $\gamma_o = (1-V^2)^{-1}$ and $H(x)$ is the Heaviside step function, and where $\omega_b^2 = 4\pi n_o e^2 \int \gamma_o m$.

As discussed in the introduction, when $V > c/\sqrt{\varepsilon}$ so that the unperturbed beam and guide modes intersect, we expect instability (Fig. 4).

We seek to derive the general dispersion relation for this instability, in order to obtain quantitative results for the parameters of interest for the design and operation of a device to act as a broad-band amplifier. Such parameters, indicated in Fig. 4, are:

- The optimal frequency, ω_m, for which the amplification rate is largest.
- The corresponding spatial growth rate, δk_I.
- The cutoff frequency, ω_0, corresponding to the modified waveguide curve.
- The upper cutoff frequency, ω_1, of the unstable range
- The choice of values for ε, V, n etc for which $\omega_1 - \omega_m$ and/or δk_I maximize.

To derive the dispersion relation for the coupled beam-guide system of Figure 2, we consider $TM_{oo,k}$ modes (i.e., no azimuthal dependence).

The wave fields are given by

$$\vec{E}\ (r,z,\theta,t) = \left[E_z(r)\ \hat{e}_z + E_r(r)\ e_r\ \right] e^{ikz-i\omega t}\ , \tag{4}$$

$$\vec{B}\ (r,z,\theta,t) = B_\theta(r)\ \hat{e}_\theta\ \ e^{\,ikz-i\omega t}\ , \tag{5}$$

where $E_z(r)$ is the solution of the wave equation

$$\left\{\frac{1}{r}\ \frac{d}{dr}\ \left(r\ \frac{d}{dr}\right) - \left[k^2 - \varepsilon(r)\ \frac{\omega^2}{c^2}\right] h^2(\omega,k,r)\ \right\} E_z(r) = 0, \tag{6}$$

with E_r and B_θ given by:

$$E_r(r) = \frac{ik}{\varepsilon(r)\,\omega^2/c^2 - k^2} \frac{d}{dr} E_z(r) \quad , \tag{7}$$

$$H_\theta = \frac{\omega}{ck} D_r \quad , \tag{8}$$

and $D_r = \varepsilon E_r$, $H_\theta = B_\theta/\mu$, with μ being the magnetic permittivity. The quantity $h^2(\omega,k,r)$ is the plasma dielectric function

$$h^2(\omega,u,r) = 1 - \frac{4\pi e^2}{m\,\varepsilon(r)} \int \frac{dp_z\, f_0(v,p_z)}{\gamma^3\,(\omega - kv_z)^2} \quad , \tag{9}$$

where the equilibrium distribution function $f_0(r,p_z)$ given by Eq.3 will be assumed.

Using Eqs. (4) - (9), the dispersion relation is obtained by applying the boundary conditions:

$$\begin{aligned} &E_z(r_4) = 0 \ , \\ &\left[E_z(r)\right] = \left[B_\theta(r)\right] = 0 \ (r = r_1, r_2, r_3) \quad , \end{aligned} \tag{10}$$

where $\left[\psi(r)\right]$ indicates the jump in quantity $\psi(r)$ across the interface at r. The seven conditions (10) yield the dispersion relation.

$$0 = \begin{vmatrix} -I_0(k_o r_1) & I_0(k_o h r_1) & K_0(k_o h r_1) & 0 & 0 & 0 & 0 \\ -I_1(k_o r_1) & hI_1(k_o h r_1) & -hK_1(k_o h r_1) & 0 & 0 & 0 & 0 \\ 0 & I_0(k_o h r_2) & K_0(k_o h r_2) & -I_0(k_o r_2) & -K_0(k_o r_2) & 0 & 0 \\ 0 & hI_1(k_o h r_2) & hK_1(k_o h r_2) & -I_1(k_o r_2) & K_1(k_o r_2) & 0 & 0 \\ 0 & 0 & 0 & -I_0(k_o r_3) & -K_0(k_o r_3) & J_0(k_E r_3) & Y_0(k_E r_3) \\ 0 & 0 & 0 & -k_E I_1(k_o r_3) & k_E K_1(k_o r_3) & \varepsilon k_o J_1(k_E r_3) & \varepsilon k_o Y_1(k_E r_3) \\ 0 & 0 & 0 & 0 & 0 & J_0(k_E r_4) & Y_0(k_E r_4) \end{vmatrix} \quad (11)$$

where

$$k_o^2 = k^2 - \omega^2/c^2 \quad , \quad (12a)$$

$$k_E^2 = \varepsilon\, \omega^2/c^2 - k^2 \quad , \tag{12b}$$

$$h^2 = 1 - \frac{1}{\gamma_o^3} \left(\frac{\omega_b}{\omega - kv_o} \right)^2 \quad , \tag{12c}$$

the last expression being the dielectric for a cold beam, $f_o \sim \delta(p_z - p_o)$ and where J_m, Y_m (I_m,K_m) are the ordinary (modified) Benel functions of order m. Equation (11) is the central result of the analysis. It must be solved numerically, giving the wavenumber (Re k) and spatial growth rate (- Im k) for real ω.

III. APPROXIMATE SOLUTION OF THE DISPERSION RELATION

An approximate analytic solution of Eq. (11) can be obtained in the limit where we consider a thin annular beam at the dielectric interface; i.e.,

$$r_3 - r_2 \to 0; \quad r_{21} \equiv r_2 - r_1 \ll r_1 \ . \tag{13}$$

The introduction of this limit is done not just for the sake of the simplification and tractability it introduces to the equations, but also because it corresponds to that of the most efficient operation. In fact, since the waveguide modes outside the dielectric material are proportional to the modified Benel function, their amplitude is maximum near the dielectric interface, hence, it is desirable to place the beam the closest possible to the dielectric to maximize the coupling coefficient.

Under these assumptions, Eq. (11) can be simplified to yield the dispersion relation

$$\frac{1}{k_o} \frac{I_1(k_o r_2)}{I_o(k_o r_2)} - \varepsilon \frac{1}{k_E} \left[\frac{J_1(k_E r_2)\, Y_o(k_E r_4) - Y_1(k_E r_2)\, J_o(k_E r_4)}{J_o(k_E r_2)\, Y_o(k_E r_4) - Y_o(k_E r_2)\, J_o(k_E r_4)} \right] + r_{21} h^2 = 0 \ . \tag{14}$$

From Eq. (14), we may extract several limiting cases:

(a) If either $k_E r_2 >> 1$ or $k_E r_{42} \equiv k_E(r_4 - r_2) << 1$, we have

$$\frac{J_1(k_E r_2)\, Y_o\, (k_E r_4) - Y_1(k_E r_2)\, J_o(k_E r_4)}{J_o(k_\varepsilon r_2)\, Y_o\, (k_E r_4) - Y_o(k_E r_2)\, J_o(k_E r_4)} \simeq \operatorname{ctn}\, (k_E r_{42}) \quad , \tag{15}$$

giving

$$\frac{1}{k_o}\, \frac{I_1(k_o r_2)}{I_o(k_o r_2)} - \frac{\varepsilon}{k_E} \operatorname{ctn}\, (k_E r_{42}) + r_{21} h^2 = 0 \quad . \tag{16}$$

(b) If $k_o r_2 >> 1$, then

$$\frac{I_1(k_o r_2)}{I_o(k_o r_2)} \simeq 1 - \frac{1}{2\, k_o r_2} \simeq 1 \quad , \tag{17}$$

giving

$$\frac{1}{k_o} - \frac{\varepsilon}{k_E} \operatorname{ctn}\, (k_E r_{42}) + r_{21} h^2 = 0 \tag{18}$$

Equation (17) is not valid near the cutoff ω_o. The result (18) is indentical to that obtained in planar geometry; this is the limiting case of large aspect ratio.

(c) Near the beam modes, $k_E r_{42} \lesssim \pi/2$, and we have

$$\frac{1}{k_o} - \frac{\pi}{2}\, \frac{\varepsilon}{k_E} + \varepsilon\, r_{42} = -\, r_{21} h^2 \ . \tag{19}$$

<u>The cutoff frequency</u> ω_o $(k = 0)$ is obtained from

$$\frac{J_1(\omega_o r_2/c)}{J_o(\omega_o r_2/c)} = \varepsilon^{\frac{1}{2}} \text{ ctn } \left(\frac{\omega_o r_2}{c} \quad \frac{\varepsilon^{\frac{1}{2}} r_{42}}{r_2}\right) \tag{20}$$

For $\varepsilon^{\frac{1}{2}} r_{42} >> r_2$, implying $\omega_o r_2/c < 1$, we find

$$\omega_o = \frac{\pi c}{2\varepsilon^{\frac{1}{2}} r_{42}} \left(1 + \frac{r_2}{2\varepsilon r_{42}}\right)^{-1} = \frac{\pi c}{2\varepsilon^{\frac{1}{2}} r_{42}} \tag{21}$$

For $\varepsilon^{\frac{1}{2}} r_{42} \lesssim r_2$ $(\omega_o r_2/c \gtrsim 1)$, we have

$$\omega_o = 2.405 \ c/r_4 \ . \tag{22}$$

Near the beam mode, we have from Eq. (19) the form

$$(\omega - kc\beta)^2 \left[\frac{1}{k_o(\omega,k)} - \frac{\pi}{2} \quad \frac{\varepsilon}{k_E(\omega,k)} + \varepsilon r_{42}\right] = r_{21} \frac{\omega_b^2}{\gamma_o^3} \tag{23}$$

where $\beta \equiv V_o/c$.

Taking $k = \omega/\beta c + \delta k$, where $|\delta k| << |k|$,

we find

$$(\delta k)^2 D_g(\omega,k) = \frac{r_{21}\omega_b^2}{\gamma^3\beta^2 c^2} \ , \tag{24}$$

where

$$D_g(\omega,k) = \frac{1}{k_o} - \frac{\pi}{2}\frac{\varepsilon}{k_E} + \varepsilon r_{42} \quad . \tag{25}$$

Expanding D_g about $k = \omega/\beta c$, we have

$$(\delta k)^2 \left[D_g(\omega, \frac{\omega}{\beta c}) + \frac{\partial D_g}{\partial k} (\omega, \frac{\omega}{\beta c})\delta k\right] = \frac{r_{21}\omega^2}{\gamma^3\beta^2c^2} \quad , \tag{26}$$

which can be solved for $\delta k(\omega)$. The maximum growth rate is obtained at $\omega \equiv \omega_m$ for $D_g(\omega_m, \omega_m/\beta_c) = 0$, giving

$$\omega_m = \frac{\pi}{2} \frac{c}{r_{42}} \frac{\beta}{\sqrt{\varepsilon\beta^2 - 1}} \left[1 - \frac{2}{\pi} \frac{\gamma}{\varepsilon} \sqrt{\varepsilon\beta^2-1}\right] \tag{27}$$

as the frequency at which the growth rate maximizes; the corresponding growth rate is given by the imaginary part of δk_m, where,

$$\delta k_m^3 = - \frac{r_{21}\omega_p^2}{r_{42}^2c^2} \frac{\pi}{2} \frac{\sqrt{\varepsilon\beta^2-1}}{\varepsilon\beta^2\gamma^3} \frac{\left[1 - \frac{2}{\pi}\frac{\gamma}{\varepsilon} \sqrt{\varepsilon\beta^2-1}\right]^2}{\left[1 + \frac{2}{\pi}\frac{\gamma^3}{\varepsilon} (\varepsilon\beta^2-1)^{3/2}\right]} \tag{28}$$

From Eqs. (27) and (28) we deduce an important constraint, <u>viz</u>, that the spatial growth rate and frequency bandwidth are universely related. Large growth rates can be obtained at the expense of limited bandwidth, and vice versa. To see this, we note that $|(\delta k_m)^3|$ maximize for

$$\varepsilon >> 1; \ \varepsilon\beta^2 \to 2 \ , \tag{29}$$

where we set $\gamma \sim 1$ (i.e., the beam is only weakly relativistic). In this limit, we find

$$\omega_m \simeq \frac{\pi c}{2r_{42}} \frac{\beta}{\sqrt{\varepsilon\beta^2-1}} \quad , \tag{30}$$

and

$$(\delta k)_m = \frac{1}{2}\left(1 - i\sqrt{3}\right)\frac{1}{\gamma}\left[\frac{r_{21}\omega_b^2}{r_{42}^2 c^2}\ \frac{\pi}{2}\ \frac{(\varepsilon\beta^2 - 1)^{\frac{1}{2}}}{\varepsilon\beta^2}\right]^{1/3} \quad . \tag{31}$$

Thus in the limit $\varepsilon^{\frac{1}{2}} r_{42} > r_2$ (for $\varepsilon >> 1$), we have

$$\frac{\omega_{on}}{\omega_o} = \sqrt{\frac{\varepsilon\beta^2}{\varepsilon\beta^L - 1}} = \sqrt{2} \tag{32}$$

under the conditions (29) for which $|\delta k_m|$ is maximum.

On the other hand, the bandwidth $\delta\omega$ is given by the upper cutoff frequency ω_1, where Im$\delta k = 0$, as

$$\delta\omega = \omega_1 - \omega_m = \frac{3}{2\gamma}\left[\frac{c r_{21}\omega_b^2}{r_{42}^2}\ \frac{\pi\beta}{\varepsilon}\ (\varepsilon\beta^2 - 1)^{-5/2}\right]^{1/3} \tag{33}$$

Thus from (28) and (33) we find the very useful relation

$$\frac{\delta\omega}{c\,\mathrm{Im}\delta k_m} = 2.18\ \frac{\beta}{\varepsilon\beta^2 - 1} \quad , \tag{34}$$

which yields a maximum for $\delta\omega$ at

$$\varepsilon \to 2, \quad \varepsilon\beta^2 \to 1, \ \gamma \to \sqrt{2} \quad ; \tag{35}$$

the regime (35) is seen to be the opposite limit to that of relation (29).

IV. DISCUSSION AND CONCLUSIONS

We shall briefly discuss the consequences of relaxing a simplifying assumption we have used in the analysis, viz that the beam is taken to be cold and uniform in density. We have neglected effects of temperature and energy shear. Across the beam, the wave field is given approximately (for $r_{21} << r_1$) by

$$\left[\frac{d^2}{dr^2} - k_o^2 \; h^2 \; (r)\right] E_z(r) = 0 \quad , \tag{36}$$

where

$$h^2(r) = 1 - \frac{4\pi e^2}{m} \int \frac{f_o(r,p) \; dp}{\gamma^3(\omega - kv)^3} \tag{37}$$

If h(r) is a smooth function, Eq. (36) has the WkB solution

$$E_z(r) = h^{-1/2} \exp \left(k_o \int h(r) \; dr\right) \quad . \tag{38}$$

We may estimate the effects of energy spread and self fields (manifested in the potential $\phi(r)$, by taking an equilibrium distribution of the form

$$f_o\left(r, p\right) = \frac{1}{\sqrt{\pi m} \; E_{th}} \sqrt{\frac{p^2}{2m} - e\phi(r)} \quad \exp\left[- \frac{(p^2/2m - e\phi(r) - E_o)^2}{2 \; E_{th}^2}\right] \; , \tag{39}$$

where E_o is the nominal beam energy and E_{th} is a measure of thermal energy spread. Then the quantity $\omega_b^2/\gamma^3(\omega - kv)^2$ used for the analysis of a cold beam is to be replaced by

$$\frac{\omega_{po}^2}{\gamma_o^3 (\omega - kv_o)^3} \left\{ 1 - \sqrt{\frac{2}{mE_o}} \frac{ek\delta\phi}{\omega_{bo}} + \frac{3k^2}{2m(E_o + e\phi_o)} \frac{E_{th} + e^2\delta\phi^2}{\omega_{bo}^2} \right\} \tag{40}$$

where quantities subscripted "zero" are evaluated at the center of the beam and where

$$e\delta\phi = m\omega^2{}_{bo}\ r_{21}^2/8 \tag{41}$$

is the potential energy drop between the edge and the center of the beam. The energy shear associated with the self-fields has two effects:

(i) The effective beam density ω_{bo}^2 is reduced:

$$\omega_{bo}^2 \rightarrow \omega_{bo}^2 \left(1 - \frac{ek\delta\phi}{\sqrt{2mE_o}\,\omega_b}\right)^2 \quad ; \tag{42}$$

(ii) The effective energy spread is increased:

$$E_{th} \rightarrow \left[E_{th}^2 + \frac{2}{3} e^2 \delta\phi^2\right]^{1/2} \quad . \tag{43}$$

Summary

In the analysis of Sections II and III we have demonstrated the validity of the dielectric-loaded Cerenkov wide-band amplifier concept. We have obtained approximate relations for the growth rate and bandwidth, revealing an important tradeoff between them, cf. Eq. (34), which must be considered in design applications.

An important extension of this work would be to consider a nonlinear calculation in order to elucidate the saturation mechanism (which we expect to be beam trapping) and ultimate power levels.

FIGURE CAPTIONS

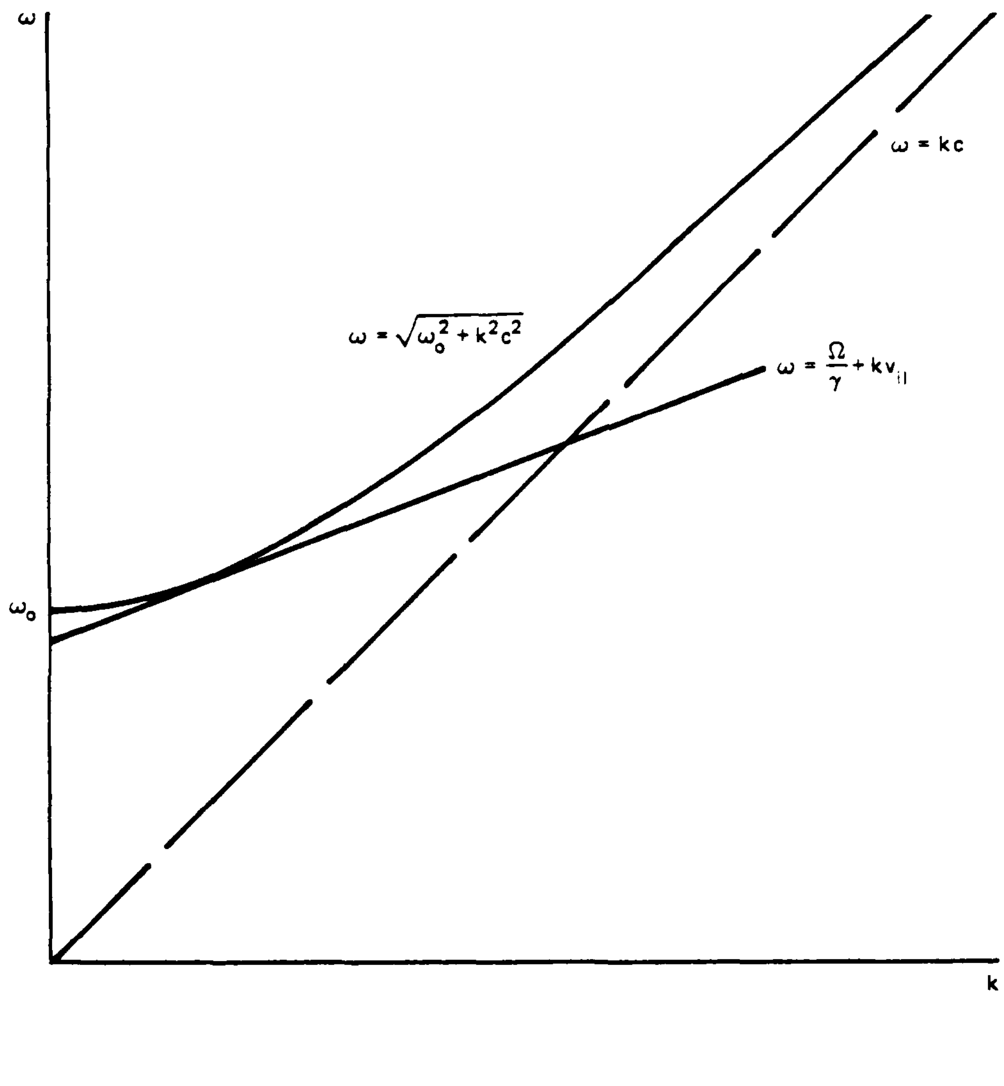
Figure 1 - Dispersion curves for the waveguide mode and the electron cyclotron mode in the electron cyclotron maser.

Figure 2 - Geometry of the Broadband Amplifier.

Figure 3a- Dispersion curves for the waveguide mode and the beam modes when $\varepsilon = 1$.

Figure 3b - Dispersion curves for the waveguide mode and the beam modes when the waveguide is filled with a dielectric material with $\varepsilon > (c/V)^2$. The broken curve refers to a partially filled waveguide.

Figure 4 - Wavenumber, Re(k), and spatial growth rate, Im(k) for the instability (schematically). The solid curves indicate the unamplified propagating modes.



ω
ω = kc
$\omega = \sqrt{\omega_o^2 + k^2c^2}$
$\omega = \frac{\Omega}{\gamma} + kv_{\parallel}$
ω_o
k

Figure 1

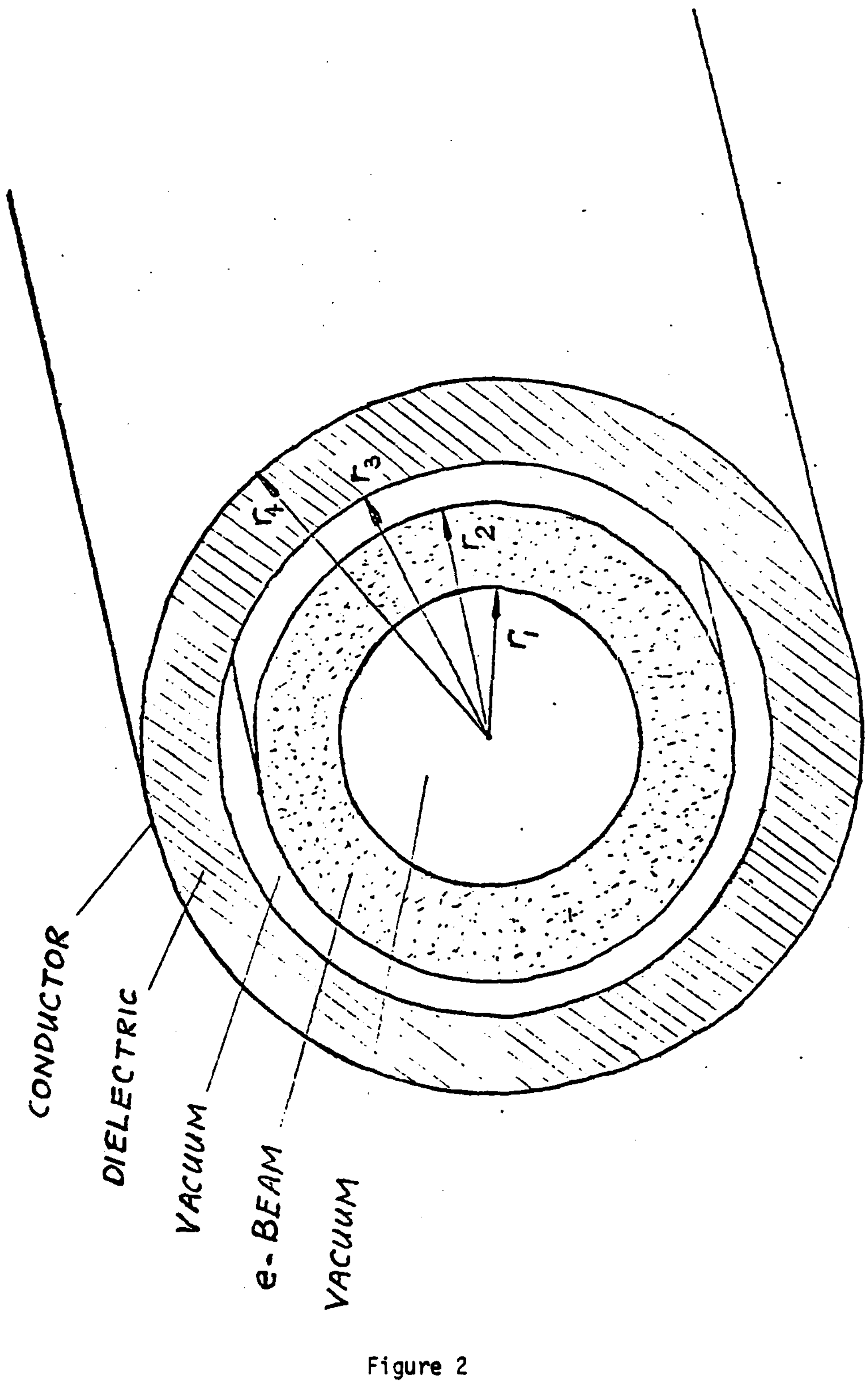


Figure 2

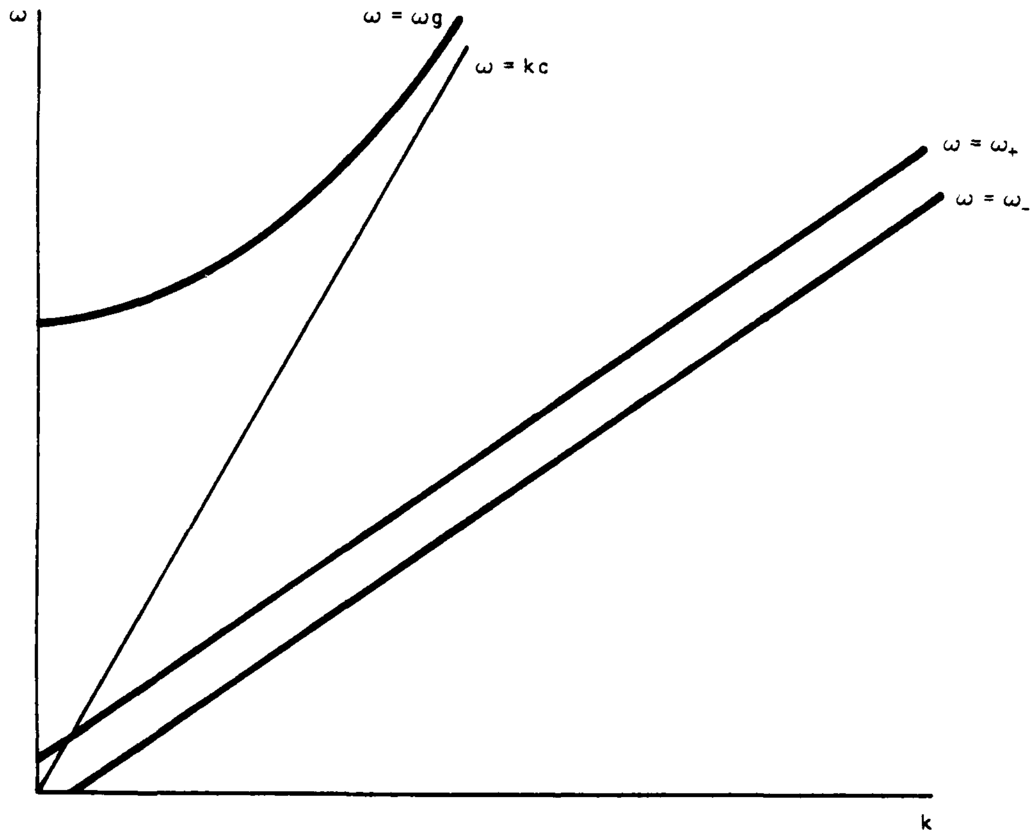


Figure 3a

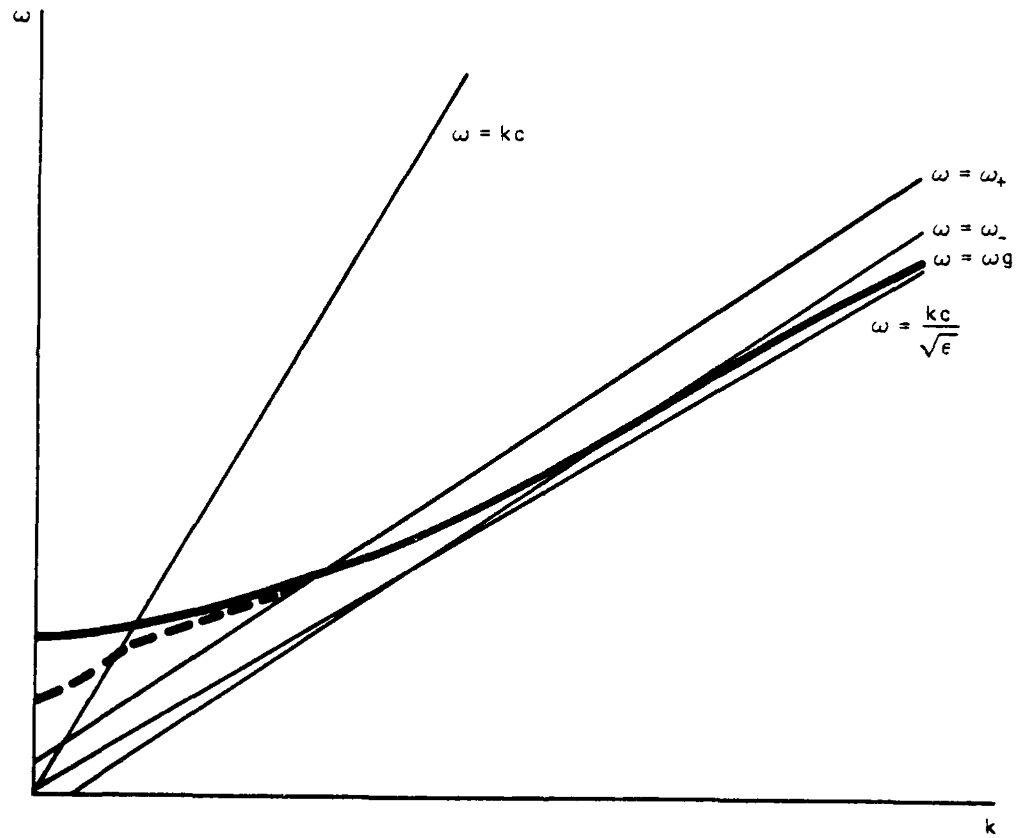


Figure 3b

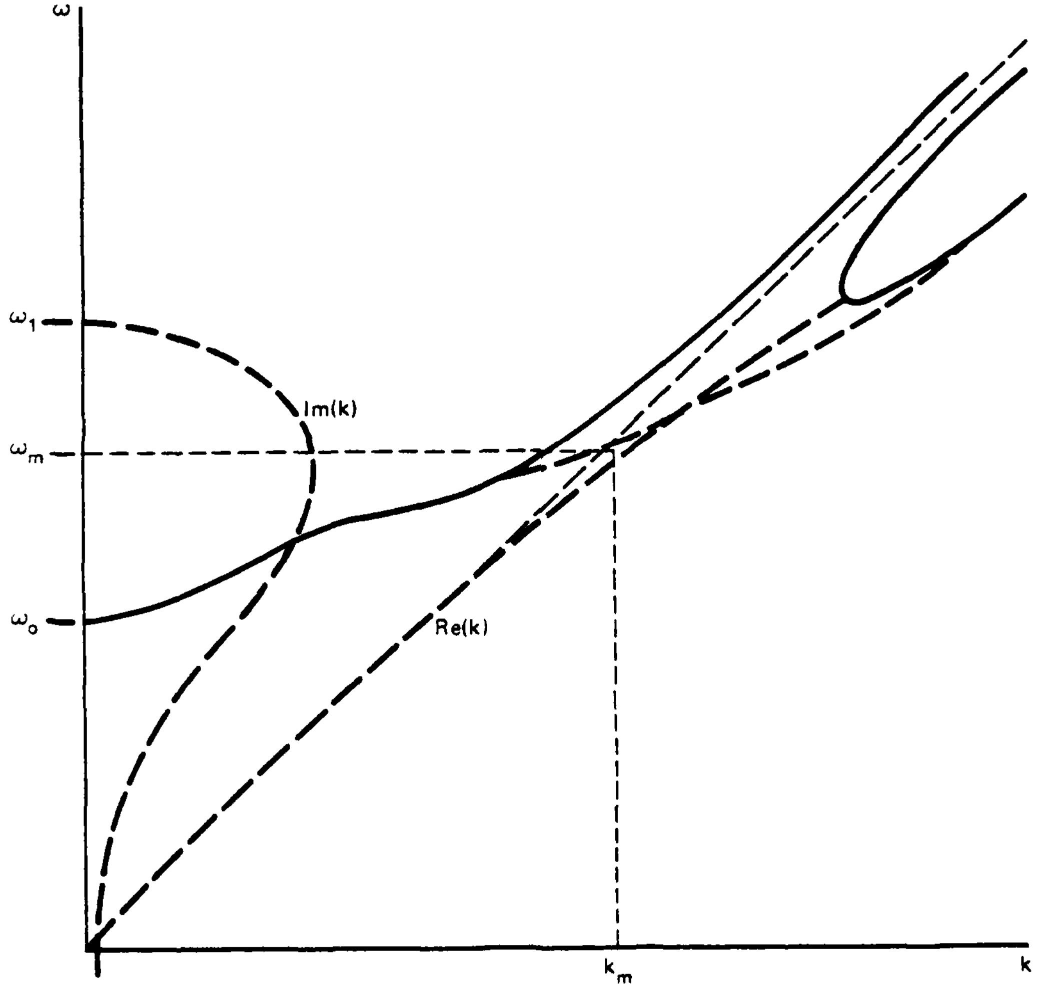


Figure 4

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